

Name:

Solutions

6 pts

1. Determine whether the function  $f(x) = x^2 - 5x - 10$  satisfies the hypotheses of the Mean Value Theorem on the interval  $[-1, 2]$ . If it does, then find the value of  $c$  that

satisfies the equation  $f'(c) = \frac{f(b) - f(a)}{b - a}$ .

$f(x)$  is cont. on  $[-1, 2]$

$$f'(x) = 2x - 5$$

$f(x)$  is differ. on  $(-1, 2)$

$$m = \frac{f(2) - f(-1)}{2 - (-1)} = \frac{2^2 - 5(2) - 10 - ((-1)^2 - 5(-1) - 10)}{3}$$

$$m = \frac{4 - 10 - 10 - 1 - 5 + 10}{3} = \frac{-12}{3} = -4$$

$$2x - 5 = -4 \quad 2x = 1 \quad x = \frac{1}{2} \quad c = \frac{1}{2}$$

4 pts

2. Determine whether the function  $f(x) = \frac{\sin x}{x}$  satisfies the hypotheses of the Mean Value Theorem on the interval  $[-\pi, \pi]$ . If it does, then find the value of  $c$  that satisfies

the equation  $f'(c) = \frac{f(b) - f(a)}{b - a}$ .

$f(x)$  is not continuous on  $[-\pi, \pi]$  since  $f'(0) = \text{undefined}$   
Therefore, the function  $f(x)$  does not satisfy hypothesis I of the MVT.

4pts

3. Determine whether the function  $f(x) = |x-1|$  satisfies the hypotheses of the Mean Value Theorem on the interval  $[0, 3]$ . If it does, then find the value of  $c$  that satisfies the

equation  $f'(c) = \frac{f(b) - f(a)}{b - a}$ .

$$f'(x) = \begin{cases} -1 & \text{if } x < 1 \\ 1 & \text{if } x > 1 \end{cases}$$

$f(x)$  is cont. on  $[0, 3]$

$f(x)$  is not diff. at  $x=1$  since the graph of  $f(x)$  has a corner at  $x=1$  or since  $\lim_{x \rightarrow 1^-} f'(x) \neq \lim_{x \rightarrow 1^+} f'(x)$   
 since  $\lim_{x \rightarrow 1^-} f'(x) = -1$  and  $\lim_{x \rightarrow 1^+} f'(x) = 1$

$f(x)$  does not satisfy hypothesis II of the MVT

7pts

4. Determine the interval(s) on which the function  $f(x) = x^2 - 10x + 1$  is (a) increasing and (b) decreasing.

$$f'(x) = 2x - 10 \quad 0 = 2x - 10 \quad x = 5 \quad f'(5) = 0$$



(a)  $f(x)$  is increasing on  $(5, \infty)$  since  $f'(x) > 0$

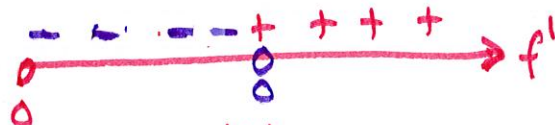
(b)  $f(x)$  is decreasing on  $(-\infty, 5)$  since  $f'(x) < 0$

7pts

5. Determine the interval(s) on which the function  $f(x) = \ln x$  is (a) increasing and (b) decreasing.

$$f'(x) = \frac{1}{x} \quad f'(0) = \text{undefined}$$

Domain of  $f(x)$  is  $(0, \infty)$



(a)  $f(x)$  is increasing on  $(0, \infty)$  since  $f'(x) > 0$

(b)  $f(x)$  is decreasing on  $(-\infty, 0)$  since  $f'(x) < 0$

7pts

6. Determine the interval(s) on which the function  $f(x) = e^{8x-x^2}$  is (a) increasing and (b) decreasing.

$$f'(x) = (8-2x)e^{8x-x^2} \quad 0 = (8-2x)e^{8x-x^2} \quad x = 4$$

$$f'(4) = 0$$



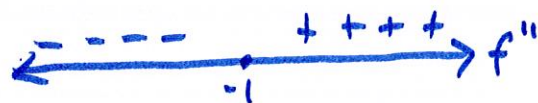
(a)  $f(x)$  is increasing on  $(-\infty, 4)$  since  $f'(x) > 0$

(b)  $f(x)$  is decreasing on  $(4, \infty)$  since  $f'(x) < 0$

- 8pt 7. Find all the point(s) of inflection of the function  $f(x) = x^3 + 3x^2 - 9x + 3$

$$f'(x) = 3x^2 + 6x - 9 \quad f''(x) = 6x + 6 \quad 0 = f''(x)$$

$$0 = 6x + 6 \quad x = -1 \quad f''(-1) = 0$$

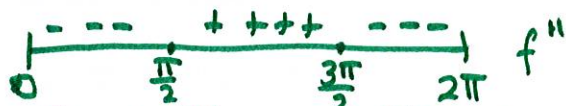


$f(x)$  has a point of inflection at  $x = -1$  since  $f'(-1) = -12$ , therefore,  $f(x)$  has a tangent at  $x = -1$  and since  $f''(x)$  changes sign at  $x = -1$

- 8pt 8. Find all the point(s) of inflection of the function  $f(x) = \cos x$  on  $[0, 2\pi]$

$$f'(x) = -\sin x \quad f''(x) = -\cos x \quad f''(x) = 0$$

$$0 = -\cos x \quad x = \frac{\pi}{2} \text{ \& } x = \frac{3\pi}{2} \quad f''(\frac{\pi}{2}) = 0 \text{ \& } f''(\frac{3\pi}{2}) = 0$$



$f(x)$  has points of inflection at  $x = \frac{\pi}{2}$  &  $x = \frac{3\pi}{2}$  since  $f'(\frac{\pi}{2}) = -1$  and  $f'(\frac{3\pi}{2}) = 1$  and since  $f''(x)$  changes sign at both  $x = \frac{\pi}{2}$  and  $x = \frac{3\pi}{2}$

- 9pts 9. Use the Concavity Test to determine the interval(s) on which the graph of the function  $f(x) = xe^x$  is (a) concave up and (b) concave down.

$$f'(x) = e^x + xe^x \quad f''(x) = e^x + e^x + xe^x = e^x(2+x)$$

$$f''(-2) = 0 \quad f'(-2) = \frac{-1}{e^2}$$



(a)  $f(x)$  is concave up on  $(-2, \infty)$  since  $f'' > 0$

(b)  $f(x)$  is concave down on  $(-\infty, -2)$  since  $f'' < 0$

- 9pts 10. Use the Concavity Test to determine the interval(s) on which the graph of the function  $f(x) = \frac{x^4}{12} - \frac{2x^3}{3} - \frac{5x^2}{2} - 6x + 7$  is (a) concave up and (b) concave down.

$$f'(x) = \frac{x^3}{3} - 2x^2 - 5x - 6 \quad f''(x) = x^2 - 4x - 5$$

$$f''(x) = 0 \quad 0 = (x-5)(x+1) \quad f''(5) = 0 \text{ \& } f''(-1) = 0$$



(a)  $f(x)$  is concave up on  $(-\infty, -1) \cup (5, \infty)$  since  $f''(x) > 0$

(b)  $f(x)$  is concave down on  $(-1, 5)$  since  $f''(x) < 0$